Multipath Array Processing for Co-Prime and Under-Sampled Sensor Arrays

David Smith
Dept. of Electrical and Computer Engineering
Duke University, Box 90291
Durham, NC 27708
phone: (919) 660-5419 fax: (919) 660-5293 email: gwh1@duke.edu

Grant Number: N00014-13-1-0201
http://people.ee.duke.edu/~jk/

LONG-TERM GOALS

Our goals are 1) the development of signal processing for co-prime sampled sensor arrays in towed array scenarios and 2) in co-prime active sonar signal designs motivated by dualities that exist between spatial arrays and temporal spectra.

OBJECTIVES

A feature of co-prime sensor arrays is that they can be designed to span large apertures with relatively few sensors. Co-prime arrays systems have been shown to be able to achieve more degrees of freedom than physical sensors present in the array. However, the cost of the achievable degrees of freedom is a loss of approximately half of the array aperture. In this project, we propose to explore and extend the synthetic aperture methodology for performance gains in co-prime and under-sampled towed array systems.

A second area of exploration in the last year has been active sonar comb signals designed with co-prime frequency spacing. Sparse frequency sampling in a co-prime comb leads to opportunities for avoidance of interferers inside the overall bandwidth of the signal, as well as bandwidth sharing with other sonar signals and increased SNR. The tradeoffs between the reduced total bandwidth of co-prime signals and the increased occupied bandwidth (relative to and equivalent comb signal) are being studied.

APPROACH

A fundamental trade-off in sensor array systems is between the cost to deploy and maintain sensors and the situational needs of array performance, such as array resolution and source localization capabilities. By non-uniformly spatially under-sampling an array, the resolution of the array is increased at the cost of increased sidelobes, sometimes referred to as grating plateaus. Synthetic Aperture (SA) methods have been applied to extend array aperture as well as impute missing sensors in sparse arrays. In addition, SA processing provides a framework to augment the sparse array in both the physical domain as well as the spatial covariance domain. This type of joint augmentation can be used to decrease side-lobe levels in sparse arrays and increase source localization performance.

Spatial covariance matrix augmentation can be used with certain classes of array structures, such as co-prime, in order to mitigate sensitivity to sources outside the mainlobe. Fully augmentable under-sampled arrays have co-arrays with no holes such that traditional uniform array processing can be used for target detection and localization. The co-array is defined as the number of times a relative spatial distance is measured by an array, which corresponds to the diagonals of a spatial covariance matrix assuming wide sense stationary data. We consider the use of SA methods for transforming the co-prime array into a synthetic fully augmentable array. Here SA is used to obtain a synthetic array that is made of up of both physical and virtual sensors such that the corresponding co-array is hole free. Thus covariance matrix augmentation can be used to increase the number of degrees of freedom of the array.

A co-prime frequency comb is a novel active sonar waveform that achieves range-Doppler performance similar to a uniform frequency comb, but uses fewer tones to do so. Several of the concepts from the co-prime array literature – notably, rank-enhanced spatial smoothing – have an frequency-domain analog. The trade-off for this reduction in occupied bandwidth is a larger bandwidth extent. Co-prime comb signals consist of tones at non-uniformly spaced frequencies according to a 2-level nested co-prime array. Specialized non-matched filter processing enables recovery of an ambiguity surface similar to that of a uniform comb, but using fewer tonal components. This reduction in occupied bandwidth offers potential benefits such as sharing, interference avoidance, and Signal-to-Noise Ratio (SNR) improvements in peak- and total-power-limited scenarios.

Key individuals at Duke that are performing this work: Juan Ramirez, Jr. (co-prime arrays) and Jon Soli (co-prime comb signals).

WORK COMPLETED

The principal challenge to using co-prime arrays (and sparse arrays in general) is to find methods to estimate the spatial covariance matrix of the co-array so that it retains the Toeplitz property and is positive definite. A subspace method called Adaptive Channel Compensation (ACC) [1], was applied to this problem. ACC was originally developed with the goal of dealing with sensor arrays with faulty sensors within the array aperture. The main idea of this technique is to use the reliable sensors along with the an estimate of the full array covariance matrix to produce an estimate of sensor readings at the missing sensors. As part of this research, we have explored applying this technique to overcome positive definite deficiency encountered from estimating the spatial covariance matrix using the direct augmentation approach (DAA). Our approach varies in that fact that the subspace of the spatial covariance matrix were are minimizing our projection onto is the that corresponding to the negative eigenvalues. The resulting optimization problem results in closed-form solution that can be used in conjunction with a subspace method like MUSIC in order to get estimates of source direction. This approach has been tested in simulation.

In the case of research on co-prime comb signal processing, we have applied correlation matrix estimation techniques in the frequency domain (in contrast to the spatial domain) to get range-Doppler estimates. Specifically, the estimation technique is a frequency-domain analog to rank-enhanced spectral smoothing popularized by Pal and Vaidyanathan [2]. The key motivation for the co-prime comb is that it occupies a smaller total bandwidth than the equivalent uniform comb signal (with the same number of tones as the "co-array" of the co-prime comb), with a greater per-tone SNR. In direct analogy to co-prime array processing, co-prime combs, in combination with an adaptive technique like MVDR allows the resolution of more targets (over an unambiguous range interval) than the number of

transmitted tones, and also enables the resolution of a weak target from a strong interferer. These ideas have been tested in simulation for a flat-fading target.

RESULTS

I. Co-Prime Array Work

An example of the ACC approach to spatial covariance estimation, consider a two source signal environment with a strong interferer at -5 degrees from broadside and a weak source at broadside. Fig. 1(a) shows the simulation results for this situation. We consider the physical co-prime array with no SA, the restricted co-prime array, the synthetic array (blue line) and a ULA (dashed red line). From the spatial power spectrum we see that the weak source is undetectable for each type of array. In Fig. 1(b), which compares the MUSIC spectra of the synthetic array and the ULA, we are able to see both sources. In addition, the MUSIC spectrum for both the synthetic array and ULA are very similar.

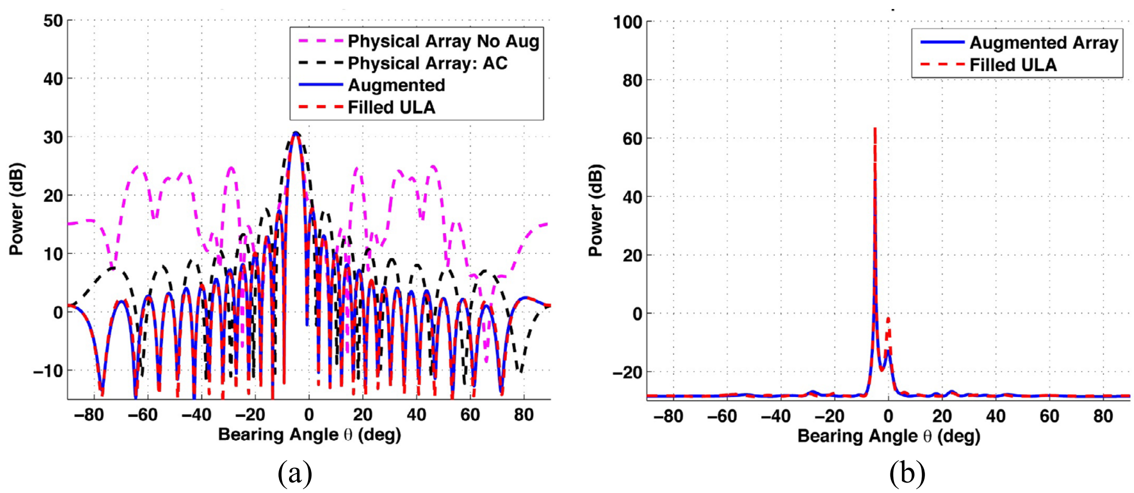
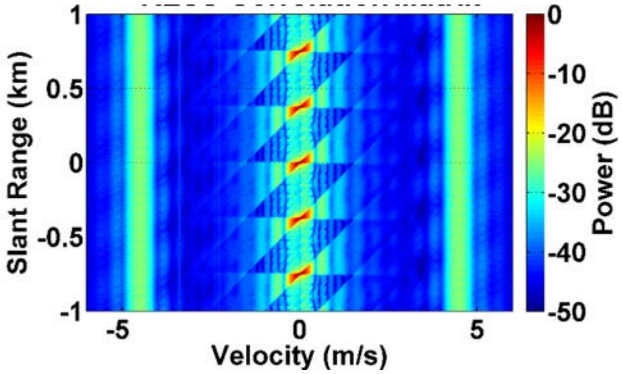


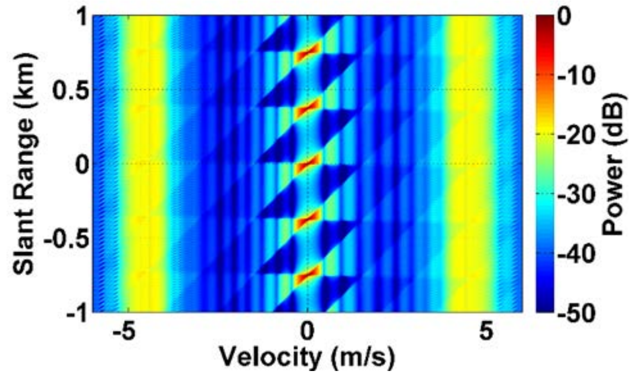
Figure 1: (a) Spatial power spectrum vs. array type, (b) MUSIC spectrum vs array type.

II. Co-Prime Comb Signals

Fig. 2(a) shows the range-Doppler response of the co-prime comb generated using rank-enhanced spectral smoothing augmentation and Fig. 2(b) shows the range-Doppler response of the conventionally processed uniform comb. Visual comparison of the two surfaces reveals that their performance is similar. Range ambiguities appear in both plots across v = 0 that matches the predicted separations of 0.375 km. The first set of velocity ambiguities are also present (smeared vertically over range) at the predicted separation of 4.478 m/s.



(a)



(b)

Figure 2: (a) Range-velocity surface for a co-prime comb signal, (b) Range-velocity surface for a uniform comb. 50 data snapshots and an input SNR of 20 dB were assumed.

MVDR range-processing results are shown in Figs. 3 and 4. In Fig. 3, a weak target is resolved in range from an interferer that is 20 dB stronger and only 5 m away from the target. This resolution is made possible by using processing the frequency domain signal return via the MVDR technique. Fig. 4 shows that MVDR processing of a scenario with 20 targets using a co-prime comb signal with only 16 frequencies is still capable of resolving. Rank-enhanced spectral smoothing enables resolving more targets than transmitted frequencies within one unambiguous range interval.

It is believed that this work with co-prime combs is novel, and work is ongoing to extend it to more challenging fading models.

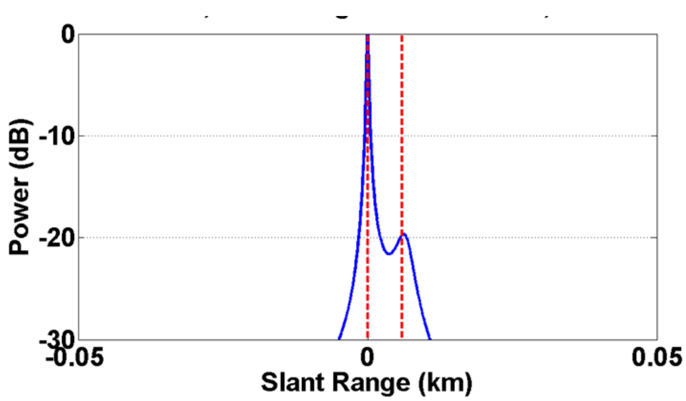


Figure 3: MVDR range processing of a weak target in the presence of a strong interferer. 50 data snapshots, an input SNR of 20 dB, and M = 5 and N = 7 were assumed.

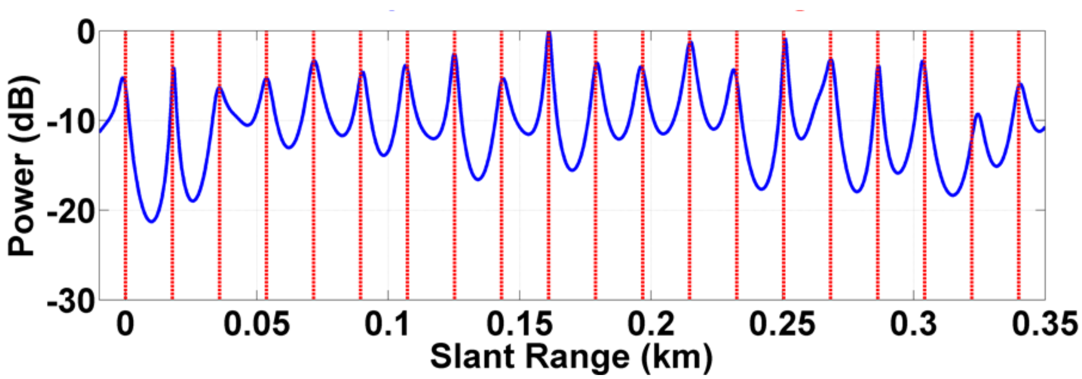


Figure 4: MVDR processing of 20 targets using a co-prime comb with only 16 frequencies. 50 data snapshots, an input SNR of 20 dB, and M = 5 and N = 7 were assumed.

Summary/Conclusion:

a) Application of Adaptive Channel Compensation to co-prime arrays, in conjunction with sub-space processing techniques, can lead to improved target localization in the presence of strong interferers.

b) Co-prime comb waveforms, believed to be novel, may achieve performance on par with an equivalent uniform comb signal.

IMPACT/APPLICATIONS

The research into SAR with co-prime arrays could result in cheaper systems, while co-prime comb signals could lead to better SNR margins while providing a degree of frequency agility to avoid in-band interferers or allow bandwidth sharing with other signals.

RELATED PROJECTS

No related projects.

REFERENCES

[1] O. Kazanci and J. Krolik, “Single-snapshot beamformer performance using large arrays with faulty sensors,” *Sensor Array and Multichannel Processing*, *Fourth IEEE Workshop on*, pp. 6-10, July 2006.

[2] P. Pal and P. Vaidyanathan, “Coprime sampling and the music algorithm,” in *Digital Signal Processing Workshop and IEEE Signal Processing Education Workshop (DSP/SPE), 2011 IEEE*, pp. 289-294, Jan 2011.

PUBLICATIONS

1. J. Ramirez and J. Krolik, “Multiple source localization with moving co-prime arrays,” in *Acoustics, Speech and Signal Processing (ICASSP), 2015 IEEE International Conference on*, 2015.

2. J. Soli and G. Hickman, “Co-prime Comb Signals for Active Sonar,” in the *2015 Underwater Acoustic Signal Processing Workshop* (accepted).